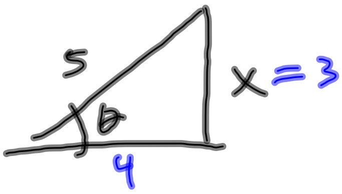
23.



$\frac{3\text{ rad}}{\text{min}} = \frac{d\theta}{dt}$ find $\frac{dx}{dt}$

when $x = 3$

$5 \sin\theta = \frac{x}{\cancel{5}}$

$\frac{dx}{dt} = 5\cos\theta \frac{d\theta}{dt} = 12$

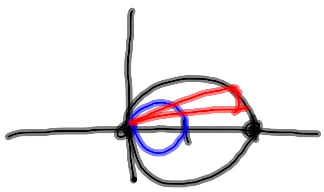
$5 \cdot \frac{4}{5} \cdot 3$

Apr 6-7:46 AM

21.

$r = 2\cos\theta$

$r = \cos\theta$



$\int_0^{\pi} \frac{1}{2}(2\cos\theta)^2 - \frac{1}{2}\cos^2\theta \, d\theta$

θ	r
0	2
$\frac{\pi}{2}$	0
π	-2

$\frac{1}{2}\int_0^{\pi} 4\cos^2\theta - \cos^2\theta \, d\theta$

$\frac{3}{2}\int_0^{\pi} \cos^2\theta \, d\theta = \cancel{2 \cdot \frac{3}{2}}\int_0^{\pi/2} \cos^2\theta$

Apr 6-9:25 AM

15.

$x = \cos^3 t \qquad y = \sin^3 t$

$\frac{dx}{dt} = -3\cos^2 t \sin t \qquad \frac{dy}{dt} = 3\sin^2 t \cos t$

$$\int_0^{\frac{\pi}{2}} \sqrt{(3\cos^2 t \sin t)^2 + (3\sin^2 t \cos t)^2}\, dt$$

$$\int_0^{\frac{\pi}{2}} \sqrt{9\cos^4 t \sin^2 t + 9\sin^4 t \cos^2 t}\, dt$$

$$\sqrt{9\sin^2 t \cos^2 t(\cos^2 t + \sin^2 t)}$$

$$\int_0^{\pi/2} 3\sin t \cos t \, dt$$

Apr 6-9:31 AM

29. $f'(x) = \sin(x^2) = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} \dots$

series for $f'(x) = ?$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} \dots$$

$$f(x) = \frac{x^3}{3} - \frac{x^7}{7 \cdot 3!} + \frac{x^{11}}{11 \cdot 5!} \dots$$

$$-\frac{1}{7 \cdot 3!} = -\frac{1}{42}$$

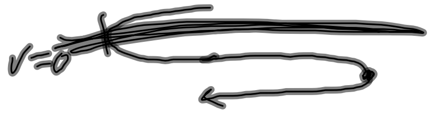
Apr 6-9:35 AM

13. $a(t) = 2t - 7$ $\quad v(0) = 6$

when farthest to right? (max position) x

$v = t^2 - 7t + c$

$6 = 0 - 0 + c$



$\frac{dx}{dt} = v = t^2 - 7t + 6 = 0$

$(t-1)(t-6) = 0$

$v = 0$

$t = 1$ $\quad t = 6$

$a(1) = -5 < 0$

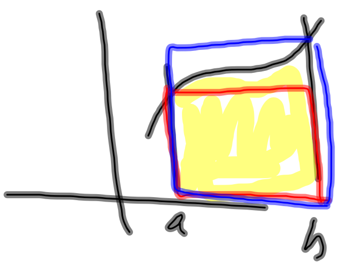
Apr 6-9:41 AM

Review 18 Prop. of def. integrals

applications of integration

area of rect $\qquad\qquad$ area of rect.

$\min f \cdot (b-a) \le \int_a^b f(x)\,dx \le \max f \cdot (b-a)$



Apr 6-9:51 AM

applications   pay attention to units

$$\int_a^b \text{rate of change}\, dt = \text{total, net, displacement}$$

rate pos (total)

rate changes sign (net)

p177 #2 $P = 1.3 + 1.025^t \quad \frac{\text{kbu}}{\text{yr}}$

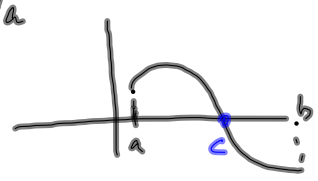
$$\int_0^{14} 1.3 + 1.025^t \, dt = \text{total production from 1990-2004}$$

Apr 6-9:55 AM

area means: total area

1. area bounded by f(x) & x-axis

$$\int_a^b f(x)\,dx = \text{net area}$$

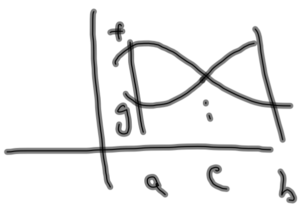


$$\text{area} = \int_a^c f(x)\,dx - \int_c^b f(x)\,dx$$

2. area between 2 curves

$$\int_a^b f(x) - g(x)\,dx \qquad f(x) \geq g(x)$$

If they cross, break it up



$$\int_a^c f - g\,dx + \int_c^b g - f\,dx$$

Apr 6-10:00 AM

Volumes of revolution

Choose: horiz (dy) or vertical (dx) rectangles (strips)

rotate the rect. (draw)

disks $\int_a^b \pi r^2\,dx$ or $\int_c^d \pi r^2\,dy$

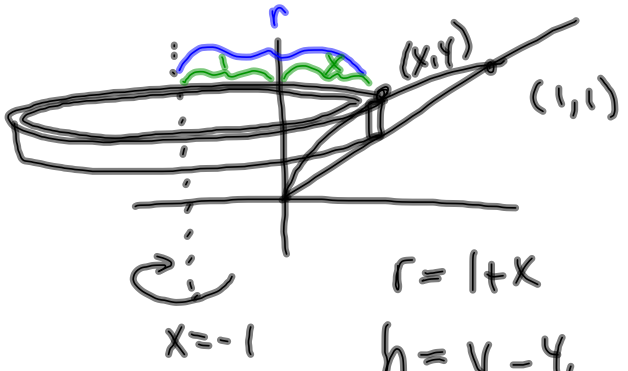
need r in terms of x — need r in terms of y

washers $\int_a^b \pi R^2 - \pi r^2\,dx$ or $\int_c^d \pi R^2 - \pi r^2\,dy$

shells $\int_a^b 2\pi r h\,dx$ or $\int_c^d 2\pi r h\,dy$

Apr 6-10:06 AM

find the volume from rotating the area bounded by $y = \sqrt{x}$, $y = x$ about the line $x = -1$



shell

$\int 2\pi r h\,dx$

$r = 1 + x$

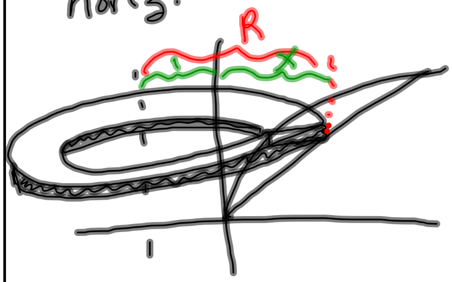
$h = y_2 - y_1$

$h = \sqrt{x} - x$

$\int_0^1 2\pi (1+x)(\sqrt{x} - x)\,dx$

Apr 6-10:12 AM

horiz. washer



$$\int_c^d \pi R^2 - \pi r^2 \, dy$$

$y = x$

$R = 1 + x_1 = 1 + y$

$r = 1 + x_2 = 1 + y^2$

$y = \sqrt{x_2}$

$y_2^2 = x_2$

$$\int_0^1 \pi (1+y)^2 - \pi \left(1+y^2\right)^2 dy$$

Apr 6-10:18 AM

volumes of known cross section

given base, slice $\quad V = \int_a^b \text{area of slice } \frac{dx}{dy}$

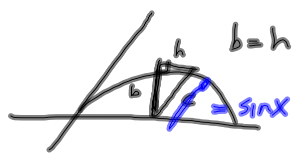
slices $\perp$ x-axis $\Rightarrow dx$

slices $\perp$ y-axis $\Rightarrow dy$

ex. base: 1 arch of $y = \sin x$

cross sections $\perp$ x-axis are right isosceles triangles with hypotenuse on xy plane

$b = h$



$$\int_0^{\pi} \tfrac{1}{2} b h \, dx$$

$$\int_0^{\pi} \tfrac{1}{2} b^2 \, dx$$

$$\int_0^{\pi} \tfrac{1}{2} \frac{\sin^2 x}{2} \, dx$$

$c = \sin x$

$b^2 + h^2 = c^2$

$2b^2 = c^2$

$b^2 = \frac{c^2}{2}$

$b = \frac{c}{\sqrt{2}}$

Apr 6-10:22 AM

arc length

$$\int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$\int_c^d \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$$

$$\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\, dt$$

Apr 6-10:29 AM